University of Salerno

Department of Civil Engineering, Conference Hall – Strength Laboratory

Ph.D. Course in *"Risk and Sustainability in Civil, Architectural and Environmental Engineering Systems"*

Professor Graeme Milton
Department of Mathematics, University of Utah (USA)
Short Course on
"Composite Materials with Extreme Mechanical Properties"

(*Section of the Course of Mathematical and Mechanical Models for Structural Engineering and Architecture***)**

Lecture 1 : July 12, 2021 - h10:00

https://zoom.us/j/98096649622?pwd=cjlUS2VpS29hd1Q5aThkUWZuZkczdz09

Lecture 2: July 14, 2021 - h10:00 ()

https://zoom.us/j/91844899457?pwd=dCt5LzgrV2dLaE51YjNENGZXTUhrUT09

Local Organizer

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Graeme W. Milton

received B.Sc. and M.Sc degrees in Physics from the University of Sydney (Australia) in 1980 and 1982 respectively. His research at Sydney University focussed on electromagnetic wave propagation through ceramic metal composites, with applications to Solar Energy. He received a Ph.D degree in Physics from Cornell University in 1985 for research in statistical physics, on models exhibiting anomalous phase transitions, and for work in composite materials showing that certain self-consistent approximations are exact for special microgeometries. He subsequently went to the Caltech Physics Department as a Weingart Fellow, from 1984 to 1986, continuing on to the Courant Institute of Mathematical Sciences as Assistant and Associate Professor of Mathematics. He is currently Professor of Mathematics at the University of Utah, Salt Lake City, Utah. His current research includes analysing the effective properties of composite materials, with particular interest in investigating new mathematical techniques which generate sharp bounds on the effective parameters, and finding the microgeometries which attain the bounds. He has studied questions relating to the electromagnetic and elastic properties of composites, and to the properties of fluid-filled porous rocks. He has held both a Sloan Fellowship and a Packard Fellowship.

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Abstract of the lectures

If one seeks a two-phase conducting structure that is optimal for guiding a maximum amount of current in a desired direction, then the answer seems intuitively obvious. Just put the high conducting phase as layers or wires in the poor conducting phase. This was proved rigorously by Raitum (1978,1993). The analogous question for elasticity is easily posed: for a given prescribed average strain what range of values can the average stress take as the microgeometry varies over all configurations? Thus, the objective is to obtain sharp bounds on the possible (average stress, average strain) pairs for two phase composites in which the volume fractions of the phases are prescribed, and to identify the optimal geometries that generate (average stress, average strain) pairs that are on the boundary of what is achievable. These optimal geometries are the most desirable for guiding stress if at the same time one wants to minimize compliance (which ensures that one is maximizing the flow of stress).

We can now answer this question for 3d-printed materials that are composites of a single elastic phase and void [1]. The optimal geometries turn out to be a particular class of pentamode structures that we call optimal pentamodes. Pentamodes are like fluids or gels in that they only support one loading; unlike fluids or gels this loading is not necessarily hydrostatic, but could be a combination of hydrostatic and shear forces. We also have a partial answer to the much grander question of identifying the set of possible elasticity tensors (including anisotropic ones) of 3d-printed materials constructed from a given elastic material with known elastic constants, and we can identify many extremal geometries with elasticity tensors on the boundary of what one can achieve. We characterize many parts of the surface of the set of possible elasticity tensors. This is no easy task as completely anisotropic 3d-elasticity tensors live in an 18-dimensional space of invariants, much more than the two invariants (bulk and shear moduli) that characterize isotropic elasticity tensors. Unfortunately, the geometries we find are rather extreme but this should motivate the search for more realistic ones that come close to having the desired elasticity tensors. Also, not all parts of the surface are characterized, even for elastically isotropic composites. Further progress will require new ideas. This is joint work with Marc-Briane, Mohamed Camar-Eddine, and Davit Harutunyan.